Optimal Control for non linear Schrödinger equations.

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Abstract

In this talk we will consider the propagation of pulses in an optical fiber with noise that is governed by the nonlinear Schrödinger (NLS) equation which in dimensionless units is

$$\partial_z u = i\partial_t^2 u + i|u|^2 u + g, \ z \in [0, \zeta], \ t \in \mathbb{R}$$

where z is the propagation distance, t is the retarded time (that is, the time in a reference frame that moves with the group velocity of the pulse), ζ is the length of the optical fiber, g describes the amplified spontaneous emission noise generation and $u(z,t)$ is the slowly varying envelope of the electric field of an optical pulse in a fiber, all quantities are in dimensionless units.

We analyze the effect of the noise on the optical fiber transmission within the framework of optimal control. We consider g as a control and we search for a control that minimizes a given cost functional having two terms involving the L^2 norm of the control and also an alternative term minimizing the distance to a given target.

In [1] we proved the existence and first order necessary conditions for a control that minimizes the given functional satisfying a restriction at the end of the line, concerning the inmunity noise level, assuming there is an additive noise in all the transmission line $g \in L^2([0,\zeta], L^2(\mathbb{R}))$. In this talk we will present these results and new ones incorporating puntual noises on the line concerning the noise generated by the amplifiers used to compensate the loss in the fiber. Also we will show some numerical simulations.

References

[1] D. Rial and C. Sánchez de la Vega. Optimal distributed control problem for cubic nonlinear Schrödinger equation. Mathematics of Control, Signals, and Systems 30 (4), 2018.

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