WELL-POSEDNESS OF INFINITE-DIMENSIONAL LINEAR SYSTEMS WITH NONLINEAR FEEDBACK

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For solutions of inhomogeneous, nonlinear partial differential equations (PDE's) we study the existence and well-posedness. The main idea is to use system theory to write the nonlinear PDE as a well-posed infinite-dimensional linear system interconnected with a static nonlinearity. Hence we assume that our nonlinear PDE has the following representation;

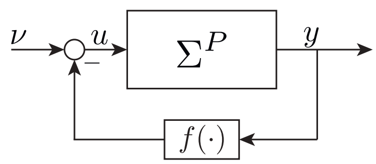

FIGURE 1. Representation of Σ^f.

Here Σ^P is a well-posed system, and f is a (static) nonlinearity, which is assumed to be (locally) Lipschitz. Let $\mathbb{F}_t$ denote the input-output map of Σ^P restricted to the time interval $[0, t]$. Our main result is the following.

Theorem 1. *If the following conditions are satisfied*

(1) *There exists $t^* > 0$ such that for all $t < t^*$, the operator $\mathbb{F}_t$ is coercive, i.e., there exists $\tilde{c} > 0$ such that for all $u \in L^2([0, t^*); U)$, it holds*
$$\langle \mathbb{F}_t u, u \rangle \geq \tilde{c} \langle u, u \rangle, \text{ for all } t < t^*,$$
(2) *$f(0) = 0$, f is continuous,*
(3) *$\forall y_1, y_2, \langle f(y_1) - f(y_2), y_1 - y_2 \rangle_U \geq 0$,*

then the nonlinear system Σ^f is well-posed and solutions exists globally.

With the use of this theorem we can show that a vibrating string with nonlinear damping is well-posed.

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