Radial basis function methods for PDEs controllability and constrained optimization problems: Recent results.

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Radial basis functions are highly effective meshfree methods for the solution of PDEs problems. These methods can be divided into global and local techniques. Although global methods have been successfully applied to several problems in different fields, recently highly successful local stable and fast methods which reassembles the classical finite difference techniques have been formulated. These techniques which are known in the literature as finite difference radial basis function, (FD-RBF), have been applied to several fields including convection-diffusion Chandhini G. (2007), Stevens D., (2009), Naiver-Stokes, Chinchapatnam P. (2009), atmospheric global electric circuit Bayona V. (2010), shallow water simulation Flyer N. (2012), reaction-diffusion on surfaces Shankar V. (2015), time-domain elastic wave propagation in 2D isotropic media Buhmann M. (2015), heat flow M. Buhmann M. (2017), among others.

Despite the successful impact of these methods in these broad fields of applied problems, these techniques are at a very preliminary stage in the field of control problems. In this talk we will first review some of the fundamental ideas behind these methods which explains its success as well as the major problems of these techniques. In particular we recall the so called uncertainty principle of Schabak which states that the condition number of the corresponding algebraic system grows, in an exponential or algebraic sense, as the number of nodes or the shape parameter increases. In this talk we review how a very recent solution to this condition number problem has been formulated providing an excellent and reliable alternative to classical methods like FD, FE or FV. More precisely, we shall review the RBF-FD poly-harmonic splines plus a high degree polynomial [1], [2], as well as the construction of hybrid kernels methods [3] which have been very recently formulated. Importantly, these new techniques makes it possible to solve large problems at different scales. Moreover, we introduce a new local symmetric and asymmetric Hermite version of these methods which makes these algorithms easer to incorporate Neumann or Robin boundary conditions.

Next, we will formulate some recent results both on controllability and optimal control problems formulated by the authors of this talk. Specifically we will talk about null controllability of the Stokes equation [4] as well as optimal control of the convection diffusion equation [5]. Both type of problems are solved by local and global methods in order to compare its relative performance. In particular the control Stokes problem is also solved by finite elements in order to compere the RBFs results with a classical method. We conclude that radial basis function methods, thought the FD-RBF poly harmonic splines and hybrid kernel techniques, provides an excellent alternative to classical methods with the advantage that no mesh is required to solve these problems. By means of several examples we show that both the numerical errors and the numerical complexity produces excellent results.

The works mentioned in this talk were done in collaboration with Jorge Zavaleta, Cristhian Montoya, Louis Breton and Christian Gout.

References:

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5. Pedro Gonzalez Casanova, Christian Gout, Jorge Zavaleta, Radial basis function methods for optimal control of the convection-diffusion equation: A numerical study, submitted, August, 2018.